New Keynesian Model

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Outline

- Private sector
- Central bank/government

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Agents:

- Households
- Final-goods producer
- A continuum of intermediate-goods producers
- Government (fiscal authority)
- Central bank (monetary authority)

Household

$$\max_{\{C_t, N_t, B_t\}_{t=1}^{\infty}} \quad \sum_{t=1}^{\infty} \beta^{t-1} \left[\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n} \right] \tag{1}$$

subject to the budget constraint

$$P_t C_t + R_t^{-1} B_t \leq W_t N_t + B_{t-1} + P_t \Phi_t + P_t T_t \tag{2}$$

C_t: Consumption, N_t: the labor supply, P_t: Price of the consumption good, W_t (w_t): nominal (real) wage, Φ_t: Profit share (dividends) of the household from the intermediate goods producers, B_t: A one-period risk free bond that pays one unit of money at period t+1, R_t^{-1}: the price of the bond. T_t is a lump-sum transfer.

Household

In real terms, the household budget constraint is

$$C_t + R_t^{-1} \frac{B_t}{P_t} \leq w_t N_t + \frac{B_{t-1}}{P_t} + \Phi_t + T_t \tag{3}$$

where $w_t = \frac{W_t}{P_t}$

Lagrange function:

$$L := \sum_{t=1}^{\infty} \beta^{t-1} \Bigg[\left(\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n} \right) - \lambda_t \left(C_t + R_t^{-1} \frac{B_t}{P_t} - w_t N_t - \frac{B_{t-1}}{P_t} - \Phi_t - T_t \right) \Bigg]$$

FONCs

$$\frac{\partial L}{\partial C_t} : C_t^{-\chi_c} - \lambda_t = 0 \tag{4}$$

$$\frac{\partial L}{\partial B_t} : -\frac{\lambda_t}{R_t P_t} + \beta \frac{\lambda_{t+1}}{P_{t+1}} = 0 \tag{5}$$

$$\frac{\partial L}{\partial N_t} : -N_t^{\chi_n} + \lambda_t w_t = 0 \tag{6}$$

Combining the first two equations, we obtain

$$\begin{aligned}
\frac{C_t^{-\chi_c}}{R_t P_t} &= \beta \frac{C_{t+1}^{-\chi_c}}{P_{t+1}} \\
\rightarrow\ C_t^{-\chi_c} &= \beta R_t C_{t+1}^{-\chi_c} \frac{P_t}{P_{t+1}} \\
\rightarrow\ C_t^{-\chi_c} &= \beta R_t C_{t+1}^{-\chi_c} \Pi_{t+1}^{-1}
\end{aligned}$$

where $\Pi_t := \frac{P_t}{P_{t-1}}$. Rearranging the third equation, we obtain

$$\begin{aligned}
& -N_t^{\chi_n} + \lambda_t w_t = 0 \\
\longrightarrow\ & -N_t^{\chi_n} + C_t^{-\chi_c} w_t = 0 \\
\longrightarrow\ & w_t = N_t^{\chi_n} C_t^{\chi_c}
\end{aligned}$$

Final-goods producer

The final good producer purchases the intermediate goods $Y_{i,t}$ at the intermediate price $P_{i,t}$ and aggregates them using CES technology to produce and sell the final good Y_t to the household and government at price P_t:

$$\max_{Y_{i,t}, i \in [0,1]} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di \tag{7}$$

subject to the CES (**C**onstant **E**lsaticity of **S**ubstitution) production function

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \tag{8}$$

Lagrange function:

$$L := P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di - \mu_t \left[Y_t - \left[\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \right]$$

$$\frac{\partial L}{\partial Y_t} : \; P_t - \mu_t = 0$$

$$\frac{\partial L}{\partial Y_{i,t}} : \; - P_{i,t} + \mu_t \frac{\theta}{\theta - 1} \left[\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di \right]^{\frac{1}{\theta-1}} \frac{\theta - 1}{\theta} Y_{i,t}^{\frac{-1}{\theta}}$$

Combining these, we obtain

$$P_t Y_t^{\frac{1}{\theta}} Y_{i,t}^{\frac{-1}{\theta}} = P_{i,t}$$

$$\Leftrightarrow \quad Y_{i,t} = \left[\frac{P_{i,t}}{P_t} \right]^{-\theta} Y_t$$

Combining the equation above with the zero-profit condition (that is, $P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di = 0$), we obtain

$$\begin{aligned} & P_t Y_t - \int_0^1 P_{i,t} \left[\frac{P_{i,t}}{P_t} \right]^{-\theta} Y_t di = 0 \\ \Leftrightarrow \quad & P_t Y_t - Y_t \int_0^1 P_{i,t} \left[\frac{P_{i,t}}{P_t} \right]^{-\theta} di = 0 \end{aligned}$$

Note that the zero profit condition is implied by perfect competition.

Dividing by Y_t,

$$\begin{aligned}
& P_t = \int_0^1 P_{i,t} \left[\frac{P_{i,t}}{P_t} \right]^{-\theta} di \\
\Leftrightarrow \quad & P_t = P_t^{\theta} \int_0^1 P_{i,t}^{1-\theta} di \\
\Leftrightarrow \quad & P_t^{1-\theta} = \int_0^1 P_{i,t}^{1-\theta} di \\
\Leftrightarrow \quad & P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di \right]^{\frac{1}{1-\theta}}
\end{aligned}$$

Intermediate-goods producers

A continuum of intermediate goods producers indexed by i:

$$\max_{P_{i,t},Y_{i,t},N_{i,t}} \sum_{t=1}^{\infty} \beta^{t-1}\lambda_t \frac{1}{P_t}\Bigg[(1+\tau)P_{i,t}Y_{i,t} - W_t N_{i,t} - P_t \frac{\varphi}{2}\left[\frac{P_{i,t}}{P_{i,t-1}} - 1\right]^2 Y_t\Bigg]$$

subject to

$$Y_{i,t} = \left[\frac{P_{i,t}}{P_t}\right]^{-\theta} Y_t, \quad Y_{i,t} = N_{i,t}$$

λ_t is the Lagrange multiplier on the household's budget constraint at time t and $\beta^{t-1}\lambda_t$ is the marginal value of an additional profit to the household. The positive time zero price is the same across firms (i.e. $P_{i,0} = P_0 > 0$).

τ is a production subsidy (later used to make the steady state "efficient").

$P_t \frac{\varphi}{2}\left[\frac{P_{i,t}}{P_{i,t-1}} - 1\right]^2 Y_t$: Quadratic price adjustment costs.

Interpretation: $\frac{\varphi}{2}\left[\frac{P_{i,t}}{P_{i,t-1}} - 1\right]^2$ is the proportion of the aggregate final goods firms would have to purchase if the firm wants to change its price from yesterday's price.

Lagrange function:

$$\max_{P_{i,t},Y_{i,t},N_{i,t}} \quad \sum_{t=1}^{\infty} \beta^{t-1}\lambda_t \Bigg[(1+\tau)P_{i,t}Y_{i,t} - W_t N_{i,t}$$

$$- P_t \frac{\varphi}{2}\left[\frac{P_{i,t}}{P_{i,t-1}} - 1\right]^2 Y_t$$

$$- \mu_{i,t}\left(Y_{i,t} - \left[\frac{P_{i,t}}{P_t}\right]^{-\theta} Y_t\right)$$

$$- \phi_{i,t}\left(Y_{i,t} - N_{i,t}\right)\Bigg]$$

$$\frac{\partial L}{\partial P_{i,t}}:$$

$$\frac{\lambda_t}{P_t}\left[(1+\tau)Y_{i,t}-\varphi\left(\frac{P_{i,t}}{P_{i,t-1}}-1\right)\frac{1}{P_{i,t-1}}P_tY_t-\mu_{i,t}\theta\left(\frac{P_{i,t}}{P_t}\right)^{-\theta-1}\frac{Y_t}{P_t}\right]$$
$$+\beta\frac{\lambda_{t+1}}{P_{t+1}}\varphi\left(\frac{P_{i,t+1}}{P_{i,t}}-1\right)\frac{P_{i,t+1}}{P_{i,t}^2}P_{t+1}Y_{t+1}=0$$

$$\frac{\partial L}{\partial Y_{i,t}}:(1+\tau)P_{i,t}-\mu_{i,t}-\phi_{i,t}=0$$

$$\frac{\partial L}{\partial N_{i,t}}:-W_t+\phi_{i,t}=0$$

Combining them, we obtain

$$\lambda_t \Bigg[(1+\tau) Y_{i,t} - \varphi \left(\frac{P_{i,t}}{P_{i,t-1}} - 1 \right) \frac{P_t}{P_{i,t-1}} Y_t$$

$$+ \left(W_t - (1+\tau) P_{i,t} \right) \theta \left(\frac{P_{i,t}}{P_t} \right)^{-\theta-1} \frac{Y_t}{P_t} \Bigg]$$

$$+ \beta \frac{\lambda_{t+1} P_t}{P_{t+1}} \varphi \left(\frac{P_{i,t+1}}{P_{i,t}} - 1 \right) \frac{P_{i,t+1}}{P_{i,t}^2} P_{t+1} Y_{t+1} = 0$$

Imposing that (i) the time zero price is the same across firms (i.e. $P_{i,0} = P_0 > 0$) and that (ii) prices are the same across firms for all time $t > 0$ ($P_{i,t} = P_{j,t} = P_t$, and thus $Y_{i,t} = Y_{j,t} = Y_t$, $\forall i \neq j$),

$$\lambda_t \left[(1+\tau) Y_t - \varphi \left(\Pi_t - 1 \right) \Pi_t + \left(W_t - (1+\tau) P_t \right) \theta \frac{Y_t}{P_t} \right] + \beta \frac{\lambda_{t+1}}{\Pi_{t+1}} \varphi \left(\Pi_{t+1} - 1 \right) \frac{P_{t+1}}{P_t^2} P_{t+1} Y_{t+1} = 0$$

Eventually, we obtain

$$Y_t C_t^{-\chi_c} \left[\varphi \left(\Pi_t - 1 \right) \Pi_t - (1+\tau)(1-\theta) - \theta w_t \right] = \beta Y_{t+1} C_{t+1}^{-\chi_c} \varphi \left(\Pi_{t+1} - 1 \right) \Pi_{t+1}$$

Market clearing conditions

The market clearing conditions for the final good, labor and government bond are given by

$$Y_t = C_t + \int_0^1 \frac{\varphi}{2}\left[\frac{P_{i,t}}{P_{i,t-1}} - 1\right]^2 Y_t di \tag{9}$$

$$N_t = \int_0^1 N_{i,t} di \tag{10}$$

Private-sector equilibrium

Given P_0 and a policy instrument $\{R_t\}_{t=1}^{\infty}$, an equilibrium consists of allocations $\{C_t,\ N_t,\ N_{i,t},\ Y_t,\ Y_{i,t}\}_{t=1}^{\infty}$, prices $\{W_t,\ P_t,\ P_{i,t}\}_{t=1}^{\infty}$ such that (i) given the determined prices and policies, allocations solve the problem of the household, (ii) $P_{i,t}$ solves the problem of firm i, and (iii) all markets clear.

$\{C_t, Y_t, N_t, \Pi_t, w_t, R_t\}$:

$$C_t^{-\chi_c} = \beta R_t C_{t+1}^{-\chi_c} \Pi_{t+1}^{-1} \tag{11}$$

$$w_t = N_t^{\chi_n} C_t^{\chi_c} \tag{12}$$

$$\begin{aligned} & \frac{Y_t}{C_t^{\chi_c}} \left[\varphi \left(\Pi_t - 1\right) \Pi_t - (1-\theta) - \theta(1-\tau) w_t\right] \\ = & \beta \frac{Y_{t+1}}{C_{t+1}^{\chi_c}} \varphi \left(\Pi_{t+1} - 1\right) \Pi_{t+1} \end{aligned} \tag{13}$$

$$Y_t = C_t + \frac{\varphi}{2} \left[\Pi_t - 1\right]^2 Y_t \tag{14}$$

$$Y_t = N_t \tag{15}$$

Outline

- Private sector
- Central bank/government

Government (Fiscal Authority)

The supply of the government bond (B_t^g) is zero. The market clearing condition for the bond is given by

$$B_t = 0. \tag{16}$$

The government budget constraint is given by

$$P_t T_t + \tau p_t y_t = 0 \tag{17}$$

This equilibrium condition only determines T_t and does not affect other parts of the model.

Central Bank

Three cases:

- CB follows an interest-rate feedback rule.
- CB optimizes under commitment (Ramsey policy)
- CB optimizes under discretion (Markov-perfect policy)

Interest-rate feedback rule

CB follows an interest-rate feedback rule

Interest-rate feedback rule

Economists often assume that the central bank is following a particular interest-rate feedback rule.

- Easier to work with.
- Easier to communicate the results with non-experts.

Below is a list of rules that are often considered in policy debates:

- Taylor rule
  - $R_t = \max\left[1, \frac{1}{\beta}\Pi_t^{\phi_\pi}\right]$
- Inertial Taylor rule
  - $R_t = \max\left[1, \frac{1}{\beta}R_{t-1}^{\rho_r}\Pi_t^{(1-\rho_r)\phi_\pi}\right]$
- Price-level targeting
  - $R_t = \max\left[1, \frac{1}{\beta}\left[\frac{P_t}{P^*}\right]^{\phi_p}\right]$
- Nominal-income targeting
  - $R_t = \max\left[1, \frac{1}{\beta}\left[\frac{P_t Y_t}{P^* Y_{ss}}\right]^{\phi_n}\right]$

Taylor-rule equilibrium

$\{C_t, Y_t, N_t, \Pi_t, w_t, R_t\}$:

$$C_t^{-\chi_c} = \beta R_t C_{t+1}^{-\chi_c} \Pi_{t+1}^{-1} \tag{18}$$

$$w_t = N_t^{\chi_n} C_t^{\chi_c} \tag{19}$$

$$\frac{Y_t}{C_t^{\chi_c}} \left[\varphi \left(\Pi_t - 1\right) \Pi_t - (1-\theta)(1+\tau) - \theta w_t\right]$$
$$= \beta \frac{Y_{t+1}}{C_{t+1}^{\chi_c}} \varphi \left(\Pi_{t+1} - 1\right) \Pi_{t+1} \tag{20}$$

$$Y_t = C_t + \frac{\varphi}{2} \left[\Pi_t - 1\right]^2 Y_t \tag{21}$$

$$Y_t = N_t \tag{22}$$

$$R_t = \max\left[1, \frac{1}{\beta} \Pi_t^{\phi_\pi}\right] \tag{23}$$

CB optimizes under commitment

- a.k.a. "Optimal commitment policy," "Ramsey policy"

Optimal commitment policy

The optimization problem of the central bank with commitment at the beginnig of time one is

$$\max_{\{C_t, Y_t, N_t, w_t, \Pi_t, R_t\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n} \right] \tag{24}$$

subject to the private-sector equilibrium conditions for all $t \geq 1$.

- The Ramsey equilibrium is defined as $\{C_t, Y_t, N_t, w_t, \Pi_t, R_t\}_{t=1}^{\infty}$ that solves this otpimization problem.
- Note that the central bank optimizes only at the beginning of time one; it does not optimize each period.

The Lagrange associated with 24 is

$$L_{RAM} = \sum_{t=1}^{\infty} \beta^{t-1} \Bigg[[\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n}]$$
$$+ \phi_{1,t} \Big[\frac{C_t^{-\chi_c}}{R_t} - \beta C_{t+1}^{-\chi_c} \Pi_{t+1}^{-1} \Big]$$
$$+ \phi_{2,t} \Big[w_t - N_t^{\chi_n} C_t^{\chi_c} \Big]$$
$$+ \phi_{3,t} \Big[\frac{Y_t}{C_t^{\chi_c}} \left[\varphi \left(\Pi_t - 1 \right) \Pi_t - (1-\theta)(1+\tau) - \theta w_t \right]$$
$$- \beta \frac{Y_{t+1}}{C_{t+1}^{\chi_c}} \varphi \left(\Pi_{t+1} - 1 \right) \Pi_{t+1} \Big]$$
$$+ \phi_{4,t} \Big[Y_t - C_t - \frac{\varphi}{2} \left[\Pi_t - 1 \right]^2 Y_t \Big]$$
$$+ \phi_{5,t} \Big[Y_t - N_t \Big] \Bigg]$$

FONCs for $t \geq 2$ are given by:

$$
\begin{aligned}
\frac{\partial L_{RAM}}{\partial C_t} &= C_t^{-\chi_c} + \phi_{1,t}(-\chi_c C_t^{-\chi_c - 1} R_t^{-1}) \\
&+ \phi_{2,t}(-\chi_c N_t^{\chi_n} C_t^{\chi_c - 1}) \\
&+ \phi_{3,t}(-\chi_c C_t^{-\chi_c - 1} Y_t \left[\varphi\left(\Pi_t - 1\right)\Pi_t - (1-\theta)(1+\tau) - \theta w_t\right]) \\
&+ \phi_{4,t}(-1) \\
&- \phi_{1,t-1}(-\chi_c C_t^{-\chi_c - 1} \Pi_t^{-1}) \\
&- \phi_{3,t-1}\left(-\chi_c C_t^{-\chi_c - 1} Y_t \varphi\left(\Pi_t - 1\right)\Pi_t\right) = 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L_{RAM}}{\partial Y_t} =& \phi_{3,t} C_t^{-\chi_c}\left[\varphi\left(\Pi_t - 1\right)\Pi_t - (1-\theta)(1+\tau) - \theta w_t\right] \\
&+ \phi_{4,t}(1 - \frac{\varphi}{2}\left[\Pi_t - 1\right]^2) + \phi_{5,t} \\
&- \phi_{3,t-1}\varphi C_t^{-\chi_c}\left(\Pi_t - 1\right)\Pi_t = 0
\end{aligned}
$$

$$\frac{\partial L_{RAM}}{\partial N_t} = -N_t^{\chi_n} + \phi_{2,t}(-\chi_n N_t^{\chi_n - 1} C_t^{\chi_c}) - \phi_{5,t} = 0$$

$$\frac{\partial L_{RAM}}{\partial w_t} = \phi_{2,t} + \phi_{3,t}(-Y_t C_t^{-\chi_c}\theta) = 0$$

$$\begin{aligned}\frac{\partial L_{RAM}}{\partial \Pi_t} =& \phi_{3,t}(Y_t C_t^{-\chi_c}\varphi(2\Pi_t - 1)) \\ &+\phi_{4,t}(-\varphi(\Pi_t - 1)Y_t) \\ &-\phi_{1,t-1}(-C_t^{-\chi_c}\Pi_t^{-2}) \\ &-\phi_{3,t-1} Y_t C_t^{-\chi_c}\varphi(2\Pi_t - 1) = 0\end{aligned}$$

$$\frac{\partial L_{RAM}}{\partial R_t} = -\phi_{1,t} C_t^{-\chi_c} R_t^{-2} = 0$$

FONCs for $t = 1$ are given by:

$$\frac{\partial L_{RAM}}{\partial C_t} = C_t^{-\chi_c} + \phi_{1,t}(-\chi_c C_t^{-\chi_c - 1} R_t^{-1})$$
$$+ \phi_{2,t}(-\chi_c N_t^{\chi_n} C_t^{\chi_c - 1})$$
$$+ \phi_{3,t}(-\chi_c C_t^{-\chi_c - 1} Y_t \left[\varphi\left(\Pi_t - 1\right)\Pi_t - (1-\theta)(1+\tau) - \theta w_t\right])$$
$$+ \phi_{4,t}(-1) = 0$$

$$\frac{\partial L_{RAM}}{\partial Y_t} = \phi_{3,t} C_t^{-\chi_c}\left[\varphi\left(\Pi_t - 1\right)\Pi_t - (1-\theta)(1+\tau) - \theta w_t\right]$$
$$+ \phi_{4,t}(1 - \frac{\varphi}{2}\left[\Pi_t - 1\right]^2) + \phi_{5,t} = 0$$

$$\frac{\partial L_{RAM}}{\partial N_t} = -N_t^{\chi_n} + \phi_{2,t}(-\chi_n N_t^{\chi_n - 1} C_t^{\chi_c}) - \phi_{5,t} = 0$$

$$\frac{\partial L_{RAM}}{\partial w_t} = \phi_{2,t} + \phi_{3,t}(-Y_t C_t^{-\chi_c} \theta) = 0$$

$$\frac{\partial L_{RAM}}{\partial \Pi_t} = \phi_{3,t}(Y_t C_t^{-\chi_c} \varphi(2\Pi_t - 1)) + \phi_{4,t}(-\varphi(\Pi - 1) Y_t) = 0$$

$$\frac{\partial L_{RAM}}{\partial R_t} = -\phi_{1,t} C_t^{-\chi_c} R_t^{-2} = 0$$

CB optimizes under discretion

- a.k.a. "Optimal discretionary policy," "Markov-perfect policy"

The time-t Lagrangean is

$$
\begin{aligned}
L_{MP,t} = {} & \frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n} + \beta V_{t+1} \\
& + \phi_{1,t}\Big[\frac{C_t^{-\chi_c}}{R_t} - \beta C_{t+1}^{-\chi_c} \Pi_{t+1}^{-1}\Big] \\
& + \phi_{2,t}\Big[w_t - N_t^{\chi_n} C_t^{\chi_c}\Big] \\
& + \phi_{3,t}\Big[\frac{Y_t}{C_t^{\chi_c}}\left[\varphi\left(\Pi_t - 1\right)\Pi_t - (1-\theta)(1+\tau) - \theta w_t\right] \\
& \qquad - \beta \frac{Y_{t+1}}{C_{t+1}^{\chi_c}} \varphi\left(\Pi_{t+1} - 1\right)\Pi_{t+1}\Big] \\
& + \phi_{4,t}\Big[Y_t - C_t - \frac{\varphi}{2}\left[\Pi_t - 1\right]^2 Y_t\Big] \\
& + \phi_{5,t}\Big[Y_t - N_t\Big]
\end{aligned}
$$

$$\frac{\partial L_{MP,t}}{\partial C_t} = C_t^{-\chi_c} + \phi_{1,t}(-\chi_c C_t^{-\chi_c - 1})$$
$$+ \phi_{2,t}(-\chi_c N_t^{\chi_n} C_t^{\chi_c - 1})$$
$$+ \phi_{3,t}(-\chi_c C_t^{-\chi_c - 1} Y_t \left[\varphi\left(\Pi_t - 1\right)\Pi_t - (1+\tau)(1-\theta) - \theta w_t\right])$$
$$+ \phi_{4,t}(-1) = 0$$

$$\frac{\partial L_{MP,t}}{\partial Y_t} = \phi_{3,t} C_t^{-\chi_c} \left[\varphi\left(\Pi_t - 1\right)\Pi_t - (1-\theta)(1+\tau) - \theta w_t\right]$$
$$+ \phi_{4,t}(1 - \frac{\varphi}{2}\left[\Pi_t - 1\right]^2) + \phi_{5,t} = 0$$

$$\frac{\partial L_{MP,t}}{\partial N_t} = -N_t^{\chi_n} + \phi_{2,t}(-\chi_n N_t^{\chi_n - 1} C_t^{\chi_c}) - \phi_{5,t} = 0$$

$$\frac{\partial L_{MP,t}}{\partial w_t} = \phi_{2,t} + \phi_{3,t}(-Y_t C_t^{-\chi_c}\theta) = 0$$

$$\begin{aligned}\frac{\partial L_{MP,t}}{\partial \Pi_t} =& \phi_{3,t}(Y_t C_t^{-\chi_c}\varphi(2\Pi_t - 1)) \\ &+ \phi_{4,t}(-\varphi(\Pi_t - 1)Y_t) = 0\end{aligned}$$

$$\frac{\partial L_{MP,t}}{\partial R_t} = -\phi_{1,t} C_t^{-\chi_c} R_t^{-2} = 0$$